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The integration and efficiency of BRICS and Pakistan stock markets: An analysis using asymmetric cointegration and MF-DFA approaches

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The integration and efficiency of BRICS and Pakistan stock markets: An analysis using asymmetric cointegration and MF-DFA approaches

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Abstract This study examines the asymmetric cointegration and efficiency between Pakistan and BRICS equity markets using monthly data from October 1997 to September 2018. The results reveal that the Brazilian stock market is the most efficient during the global financial crisis. Threshold and momentum threshold autoregressive models (TAR and M.TAR) confirm the presence of a long-run relationship between BRICS and Pakistan stock markets, where the speed of negative shocks is higher and significant for Pakistan Russian, Pakistan South Africa stock market pair. This infers quick adjustment of stock prices to negative shocks (bad news) as compared to positive shocks (good news). The speed of adjustment of positive shocks for Pakistan China is higher. The results of asymmetric error correction model (AECM) show results of unidirectional causality between Pakistan China and Pakistan Brazil stock market pairs, while bidirectional causality runs from Pakistan Russia, Pakistan India, and Pakistan South Africa stock market pairs. Thus the Pakistan stock market has short-run and long-run relationships with most other stock markets. Multi-fractal detrended fluctuation analysis supports long run efficiency of Brazilian markets during the global financial crisis. It suggests that investors pay keen attention to the Pakistan stock market when investing in BRICS stock market.

Keywords Asymmetric cointegration · MF-DFA · TAR and M.TAR · VECM · BRICS-Pakistan

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1 Introduction

A rapid growth has been noted in BRICS and Pakistan economies over the last few decades. The cointegration of BRICS and Pakistan economies gained much attention in terms of trade and investments. Goldman Sachs interprets BRIC's total nominal expected GDP to be 128 trillion dollars in 2050 as compared to world G7 countries, 66 trillion at that time. This present and future (expected) growth have a significant inference for their stock markets capitalization and also for international investors. It is reported that almost 40% of the world's market capitalization will be reported for four BRICS economies.

South Africa is the producer of deliberate commodities (Gold, Chrome, and Platinum) in the whole world, which is considered important for domestic and economic growth. South Africa can enhance the chances of formulating a more devoted investment policy with respect to portfolio diversification prospects in BRICS economies. The addition of South Africa becomes significant due to its growth at a quick pace, noteworthy development, and complexity in financial markets (Liu et al 2013; Zhang et al 2013).

Pakistan's stock market is considered economically and strategically significant. In 2017, the reinforcement of investors resulted in to better returns (upward trend) on the investment in K.S.E. (Karachi stock exchange). It is reported that on 24th May 2017 Karachi stock exchange reached its highest index points 52,876.46. A positive effect of economic factors (GDP): Gross domestic product, upgraded country insights, the establishment of SMEs (small and medium enterprises), and inflow of remittances in Pakistan are examined in the fiscal year 2018.

In the same fiscal year, 2018 (July-March) capital markets were operated in the extensive range, due to which the stock market became highly volatile. PSX index (Pakistan stock exchange) reached 47,084 index points, the topmost level on 3rd August 2017, till August 2018. On 30th March 2018 Pakistan stock exchange market capitalization was RS. 9,370.6 billion. The average daily trading value of stocks was RS. 8.54 billion and turnover per day was 192.25 million shares. The future traded volume was reported as 3.7 billion and trading as 61.4 million shares. In this period foreign investors off-loaded USD123.9 million (from July 2017 to March 2018). After this behavior, individual investors, multinational companies opted buying local securities. This buying behavior of local investors puts the confidence of investors in the Pakistan equity market. Considering this the Pakistan stock market has great significance in our sample.

Debate has increased on financial integration in pacific basin regions (Phylaktis and Ravazzolo 2002). Yang et al (2003) indicate increasing market integration between smaller markets with the US stock market for the period of 1970 to 2001, and very small to insignificant integration traces among Asian stock markets until the years of 2007-2008 (Yu et al 2010). A constant improvised negotiation and the actual initiation of the international trade phenomena, among the stock markets unlocked a potential gap for the integration phenomenon (Yao et al 2018). The growing trend of cointegration among Asian stock markets and its spillover proposed by Tiwari et al (2013) reduced the benefits of portfolio diversification for the investors in cointegrated markets (Kearney and Lucey

2004).

The diversification is completely effective for the investors when integration amid Pakistan and BRICS equity markets is least and financially sound (Zahir and Rahim 2015). Pakistan's investors are encouraged to invest in an international diversified portfolio in developed equity markets, as there exists a low cointegration between Pakistan and developed stock markets (Tiwari et al 2013). The benefits of diversification can be attained by investing in emerging and developed equity markets having low cointegration with Pakistan's stock market (Siddique et al 2016). Investors can generate arbitrage profit through diversification in the short-run in emerging economies while profit can be generated through diversification in the long run from emerging economies (Al Nasser and Hajilee 2016).

The efficiency of the Pakistan stock exchange and BRICS stock markets is examined and the inefficiency of BRICS stock markets is highlighted due to abnormal returns. A shred of visible evidence is that Brazil from all the BRICS equity markets represents a sign of efficiency in BRICS (Cheng et al., 2010 and Sajid et al.,2020). Despite the mixed results, BRICS equity markets are maintaining efficient market theory, thereby not considered as a weak form of efficiency (Tiwari et al 2013). Zahir and Rahim (2015) elucidated how and why the cointegration among the BRICS and Pakistan can be financially prosperous with insignificant integration. Joshi (2013) and Nashier (2015) focused on symmetric cointegration however An and Brown (2010) investigate the degree of cointegration between developed stock markets and BRICS.

Contrary to the previous studies, this study first contributes to employ asymmetric error correction models (VECM) to calculate asymmetric cointegration of Pakistan equity market and BRICS equity markets individually following Markowitz portfolio theory. The second efficiency of the Pakistan stock exchange and BRICS stock market is examined by using the MF-DFA model to measure the extent of the strong or weak form of efficiency. It helps investors to diversify portfolios and regulators to minimize the cost of capital for the efficient allocation of resources.

2 Literature review

Several researchers conduct studies on stock market linkages using empirical and theoretical models, there are many studies pertaining to the cointegration between stock markets. Like Shahzad et al (2015); Arshanapalli and Doukas (1993); Bessler and Yang (2003); Kasa (1992) have used cointegration techniques (cointegration analysis, VECM, etc) to examine the relationship of stock markets of developed economies. The findings of these studies provide evidence of integration among the stock markets. It also infers US stock markets have strong long-run effects on other stock markets. Other literature Narayan and Rehman (2018); Manning (2002); Arshanapalli and Doukas (1993); Yang et al (2003) of financial integration between Asian stock markets and US stock market concludes that US market has a large impact on Asian stock markets, whereas Asian stock markets do not impact the US market. The isolation of Philippines, Taiwan, and Japan stock markets indicate partial convergence of indices by the existence of a minimum of two common trends in the dataset. Similarly, research studies by Lahrech and Sylwester (2011); Chen et al (2002) found stock market integration among Latin American stock markets.

Saqib et al. (2019) examined financial cointegration and the spillover impact of the crisis on the emerging economies of India, China, Pakistan, Malaysia, Russia, and Korea. The study used JJ (Johansen and Juselius) co-integration test and V.E.C.M (Vector Error Correction Model) to observe the presence of cointegration among markets. Long-run cointegration is found among the US and the emerging markets. Cointegration increased after the financial crisis period. The results of VECM and impulse response function represents that shocks in the US markets impose a short-term effect on the returns of emerging stock markets. Korean and Bombay stock markets have a cross-market news effect and volatility spillover effect in the period of the financial crisis. There is a positive effect on Bombay and Russian stock markets while negative on Malaysian and Chinese stock markets after the financial crisis period.

Janakiramanan and Lamba (1998) who studied Asian versus developed stock market, found that all the Asian stock markets (except Indonesia) are influenced by the US stock market, however, the impact is strong on Australian stock market. In the same way in an analysis of six Asian counties, Australia China, India, and US markets; it is determined that Asian stock markets are highly cointegrated with the Chinese stock market than Australian, Indian, and US markets. Wang (2014) proposed that the global financial crisis has made the stock market of East Asia less responsive to the shocks that instigate from US markets. There exists integration between OECD, Asia, the US, and the UK stock market (Masih and Masih 2001). A unidirectional and bidirectional relationship is indicated for Asian stock markets by Narayan and Rehman (2018); Yu et al (2010); Leong and Felmingham (2003); Jang and Sul (2002). Cointegration exists among the stock indices of US, DJ DTOXX, and CSEE stock markets which is evident from the work of Boubaker and Jouini (2014).

Similarly, Stoica et al (2015) derived regional inter-dependencies among emerging stock markets and the developed market of the whole world. The author has considered two subsamples to observe the effect of financial crisis using VECM (Vector Error Correction Model). It has been concluded that Poland, the Czech Republic, and Hungary markets confirm international cointegration in the sub-sample of pre-crisis whereas Bulgaria and Romania stock markets are the more segmented markets because of less integration with international markets. In the second sub-sample period, it is reported by the author that Czech Republic, Hungary, Poland, and Romania markets are highly susceptible to the financial crisis of stock markets. The volatility of all stock markets is highly affected by the shocks that emerged from the US market except Bulgaria. The causal linkages among European emerging markets are studied by Birău and Antonescu (2014) and unidirectional causality runs from Hungary to Romania.

Using static cointegration analysis Guidi and Ugur (2014) find weak integration of South-Eastern European stock market run with German and UK markets, although these markets do not integrate with US stock markets during sub-sample of the crisis period. In the same way, there is strong existence of time-varying positive dependence between Czech, Polish, and Hungary stock markets. Romania's stock market does not integrate in the first crisis period but found integration with increased dependence after the international crisis sub-sample. In boom and bust, these markets move together for symmetric dependence however the level of integration varies across different markets during the crisis. Strong integration results in Czech and Hungry markets whereas Romania's market exhibits low integration with other stock markets. Contrary to this Poland and Hungary behave independently during extreme market movement (upward and downward).

Chittedi et al (2010) determined long-run integration between BRICS stock market indices and the developed countries (US, UK, and Japan) using Granger causality and VECM. Results report Indian stock market is highly influenced by the Japanese and US markets. UK, China, Russia, and Brazil markets do not influence the Indian market while affecting Russia and Brazil markets. Furthermore, they analyzed the integration and linkages of Indian equity markets with BRICS economies. No long-run integration is reported among BRICS countries, using JJ (Johansen and Juselius) cointegration test. There is a positive indication of cointegration (short-run bidirectional causality and long-run equilibrium) among Brazil and the Indian equity market. Brazil's stock markets are run by the Chinese market which in turn runs the Russian stock market.

From the above-discussed literature it is evident that several research studies have been performed to analyze cointegration among stock markets to see the potential sights of profitability and diversification benefits. However, the asymmetric relationship between stock needs to be explored. Hence our study comprehends the existing knowledge on inter-national stock market integration by analyzing asymmetric integration between BRICS and Pakistan stock markets which to the best of our knowledge has not been examined before.

3 Modelling Pakistan-BRICS stock market dynamics

This paper employs monthly data of Pakistan and the BRICS equity market from January 2000 to September 2018. Following are the equity index used for the analysis: Pakistan (KSE/ PSX 100), Brazil (BOVESPA), Russia (MOEX), India (BSE Sensex), China (shanghai composite), South Africa (JSE Top 40).

3.1 Unit root test

In a weak-form efficient market, if prices follow 'Random Walk' the equity market is treated as an efficient market. An efficient market quickly incorporates the external information available and shock to market endures permanently (Shahzad et al 2015). To determine the time series of stock market prices, it uses the classical unit root test. To examine a weak form of efficiency multiple unit root test is used based on the conventions. Here ADF, PP, and Shin tests are used to examine the stationarity of the series. It also examines the Random Walk Hypothesis and determines the order of integration before moving towards cointegration.

3.2 Asymmetric cointegration and causality

When there exists an asymmetric relationship between variables, there will be inaccurate symmetric cointegration. This is due to the error term used in the model having a unique speed of adjustment for a negative and positive shock. By suggesting an introduction of an error correction term in the Engle-Granger two-step model, Enders and Siklos (2001) suggested a TAR model for the asymmetric relationship among variables. Adjustments to the asymmetric relationship among variables are tested by a dual regime threshold co-integration method. We can write the equation as:

$$\Delta\varepsilon_t = l_t\rho_1\varepsilon_{t-1} + (1-l_t)\rho_2\varepsilon_{t-1} + \Sigma_{i=1}^{p}\phi_i\Delta\varepsilon_{t-i} + \mu_t \quad (1)$$

where I_t represents the indicator function. The key point in the equation is that the level ε_{t-1} is influencing indicator function. Depending upon the difference in ε_{t-1} an alternate threshold MTAR model is proposed by Enders and Siklos (2001), where, $\acute{M}_t = 1$ if $\varepsilon_{t-1} \geq 0, \acute{M}_t = 0$ if $\varepsilon_{t-1} \leq 0$. Consequently, the null hypothesis is tested as $H_o : \rho_1 = \rho_2 = 0$.

Through F-statistic the null hypothesis (cointegration) can be tested by using TAR and M-TAR models, respectively. The null hypothesis using TAR and M-TAR can be denoted as ϑ and ϑ^* respectively. Just like F-statistics, these two models have the same statistics, nonetheless, nonstandard asymptotic distribution is there. To check the null hypothesis Monte Carlo simulation was used and contributed to the value of ϑ and ϑ^* respectively (Enders and Siklos 2001). The asymmetric adjustment of H_1 although, cointegration exists in the time series is to be checked by the null hypothesis as $H_o : \rho_1 = \rho_2$.

By using the F test the null hypothesis made for the symmetric adjustment can be tested, contrary to the alternate hypothesis of asymmetric. The speed of adjustment for error correction term will be asymmetric if the variables are threshold cointegrated Y_{t-1}, when it comes above its durable equilibrium value $(\alpha_0 + \alpha_1 X_{t-1})$, then the adjustment is ρ_1, whereas, it is ρ_2, if Y_{t-1} is less than the equilibrium. The symmetric speed of adjustment described that the null hypothesis rejection chances are there. ADF cointegration presented by Engle and Granger (1987) is reflected in Enders and Siklos (2001) TAR model. The Granger asymmetric causality test can be used for the asymmetric cointegration among two-time series through asymmetric error-correction as follows:

$$\Delta y_t = \alpha_0 + \acute{\eta}_{11}\rho_1\varepsilon_{t-1} + \acute{\eta}_{12}(1-I_t)\varepsilon_{t-1} + \Sigma_{i=1}^{p}\alpha_{1i}\Delta y_{t-i} + \Sigma_{i=1}^{p}\alpha_{2i}\Delta X_{t-1} + \mu_{1t} \quad (2)$$

$$\Delta X_t = \beta_0 + \acute{\eta}_{21}I_t\varepsilon_{t-1} + \acute{\eta}_{22}(1-I_t)\varepsilon_{t-1} + \Sigma_{i=1}^{p}\beta_{1i}\Delta Y_{t-i} + \Sigma_{i=1}^{p}\beta_{2i}\Delta X_{t-1} + \mu_{2t} \quad (3)$$

where, η_{11} and η_{12} represent the adjustment speed coefficient, respectively. Similarly, $\acute{\eta}_{21}$ and $\acute{\eta}$ are adjustment speed of the coefficient of ΔX_t. The lead-lag relationship can be seen between X_t and Y_t and it is examined using the Granger Causality test.

3.3 Multifractal de-trended fluctuation analysis (MF-DFA)

In the present study Multifractal de-trended fluctuation analysis (MF-DFA) is used to rank and examine the efficiency of Pakistan and BRICS stock markets. MF-DFA proposed by Peng et al (1994) is appropriate in measuring the stock market efficiency as it allows to rank each market's efficiency individually and also indicates the degree of inefficiency (Sajid et al., 2018, Rizvi and Arshad, 2016). The assumption of linearity in the econometric model is not approved and this method experiences important trends of price patterns, which another econometric model does not incorporate (Zunino et al 2009).

Similarly, different studies explored the nature of financial time series to be multi-factual which cannot be studied by different econometric models (Mandelbrot 1997; Bacry et al 2001). The fluctuation in price pattern is determined by a spectrum of generalized Hurst exponent. Ranking of stock markets based on efficiencies using the order (index variable) Hurst exponents is being used. Following Kantelhardt et al (2002) work, MF-DFA consists of these five steps. The first step which involves the corresponding profile of correlated time series $x_i, i = 1...N$, is resolved through integration:

$$X(i) = +\Sigma_{K=1}^{i}[x_i - (x)], i = 1, ..., N \tag{4}$$

where, N represents a length of time series while x represents the mean. Afterward, settling profile, x_i is then partitioned into non-overlapping windows of equivalent length(s) $N_s = int\frac{N}{s}$. In this research, some parts in the time series may not be captured by the non-overlapping windows, as the length of the time series is not a multiple of the scale s. Consequently, attempting to ward off the data loss, they presented their work by repeating the same procedure starting from the opposite end and $2N's$ windows are gathered. This research study follows the same procedure used by Zhu et al (2011).

Afterward, the local trend is investigated for each subinterval $v = 1.....2Ns$ and each sub interval's least square fit is resolved. The distinction among the original time series and the fits creates a new series which is de-trended time series, which is as follows:

$$X_s(i) = X[(v - N_s)s + 1] - x_v(i) for\ v = N_s + 1, ...2N_s \tag{5}$$

$$X_s(i) = X[N - (v - N_s)s + 1] - x_v(i) for\ v = N_s + 1, ...2N_s \tag{6}$$

Here, v^{th} sub-interval's polynomial fit is represented as $x(v)$ and the variance is approximated as under:

$$F_{xx^2}(s, v) = \frac{1}{s}\Sigma_{v=1}^{s}X[(v - 1)s + i] - x_v(i)^2 for\ v = 1, ..., N_s \tag{7}$$

$$F_{xx^2}(s, v) = \frac{1}{s}\Sigma_{v=1}^{s}X[N - (v - 1)s + i] - x_v(i)^2 for\ v = 1, ..., N_s \tag{8}$$

Further, the variances over all sub-intervals are averaged to obtain the qth order fluctuations as:

$$F_q(s) = \frac{1}{2Ns}\Sigma_{v=1}^{2Ns}F^2(s, v)^{\frac{q}{2}^{\frac{1}{q}}} \tag{9}$$

Here, the order q can obtain any real value except 1. For $q = 0$, due to the deviating exponent, the possibility of estimating the value $h(0)$ exists. So, a logarithmic average procedure must be applied here at this point. For the assessment of $q = 2$, the ordinary DFA method is used. In the end, for every value of q, the log-log plot of $F_q(s)$ against s is considered to regulate the scaling behavior of the volatility functions. With an expansion in the scale s, $F_q(s)$ expands if time series x_t and long-range power-law interact, which is shown as: $F_q(s) \sim s^{h(q)}$

A group of scaling exponents $h(q)$ is given by the slope of the log-log of $F_q(s)$ against s. The Hurst exponent $H = h(2)$ is expressed from the scaling exponents $h(q)$. as mentioned above, the behavior of time series over time is expressed by the 'Hurst exponent'. The stock market level of inefficiency is represented as $(0 < H < 0.5 < H < 1)$ which shows a negative correlation. Afterward, $H = 0.5$ represents as not correlated Brownian motion (i.e., the efficiency of stock markets), which is perused by the time series. To calculate $h(q)$, the MF-DFA model is used, having an explicit link with the classical multifractal scaling exponent by:

$$\tau(q) = qh(q) - 1 \tag{10}$$

The spectrum and singularity strength denoted by α and $f(\alpha)$, is to be approximated by using the 'Spectrum of Generalized Hurst exponents' $h(q)$ which is shown as:

$$\alpha = h(q) + qh'(q) \; And \; f(\alpha) = q[\alpha - h(q)] + 1 \tag{11}$$

Multiple values of α in multifractal methods are used to describe different parts of the structure, which encounters the existence of spectrum $f(\alpha)$.

4 Data and analysis

This study employs a monthly dataset of Pakistan and BRICS equity markets from October 1997 to September 2018 making 258 observations of each stock market. The graph depicts a slight divergence in the movement for Russia, India, and China stock markets, whereas, high divergence is shown in Brazil, South Africa, and Pakistan's equity market. For capturing the time-varying co-movements, non-linear cointegration models are important among the stock markets as traditional linear models cannot capture the nonlinear behavior of time series (Siklos and Granger 1997).

The trend in figure 1 shows varying movements in stock markets of Pak-

Table 1: Descriptive statistics Pakistan and BRICS stock markets

	Brazil	China	India	Pakistan	Russia	South Africa
Mean	0.011	0.006	0.012	0.015	0.018	0.009
Std. Dev.	0.081	0.077	0.074	0.081	0.110	0.053
Skewness	-0.532	0.138	-0.194	-0.460	0.287	-0.675
Kurtosis	5.360	4.972	5.023	6.257	7.217	6.503
Jarque-Bera	71.787	42.478	45.461	122.750	194.050	150.990
Observations	257	257	257	257	257	257

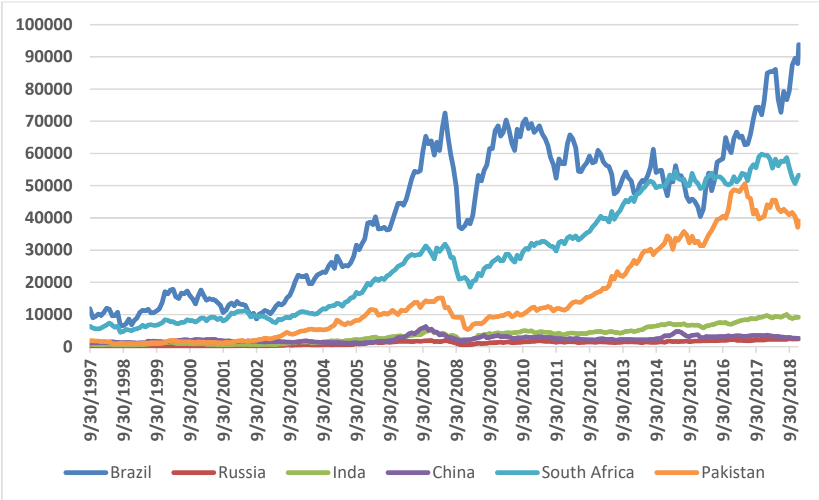

Fig. 1: Stock price trends of Pakistan and BRICS

istan and BRICS. Table 1 presents the descriptive statistics of monthly returns. Russian stock market has a high monthly average return of 1.8% and a high standard deviation of 11%. Brazil and Pakistan stock market's standard deviation is 8.1% whereas, monthly returns are 1.1% and 1.5% respectively. This shows that the Brazil stock market provides lower returns than the Pakistan stock market on a similar level of risk. Negative skewness and a high kurtosis represents fat tail distribution for the return series of Brazil, India, Pakistan, and South Africa. Jarque-Bera's significance depicts the non-normality of the time series of all stock markets.

Table 2 shows the pairwise correlation between BRCIS and Pakistan stock markets. The stock market of Pakistan and BRICS are positively correlated. Stock markets of Russia and Pakistan are highly correlated with 0.332, Pakistan and China with 0.055, Pakistan and Brazil with 0.261, India and Pakistan with 0.257 and South Africa and Pakistan with 0.148.

Table 2: Correlation among Pakistan and BRICS stock markets

	Brazil	Russia	India	China	South Africa	Pakistan
Brazil	1 ——					
Russia	0.575 0	1 ——				
India	0.481 0	0.349 0	1 ——			
China	0.275 0	0.229 0.0002	0.279 0	1 ——		
South Africa	0.611 0	0.499 0	0.468 0	0.255 0	1 ——	
Pakistan	0.261 0	0.333 0	0.257 0	0.056 0.372	0.149 0.017	1 ——

4.1 Unit root test

To check cointegration, it is necessary to examine whether either series are integrated at level one or not. Table 3 reports the results of ADF (Augmented Dickey-Fuller), PP (Philips-Perron unit root), and Shin (Kwiatkowski-Phillips-Schmidt-Shin). Trend and intercept have the same results as non-stationary except China. However, at first difference time series becomes stationary, rejecting the null hypothesis for Pakistan and BRICS stock markets.

Table 3: Unit root test: ADF, PP, and Shin

	Series	Brazil	Russia	India	China	South Africa	Pakistan
ADF	Level	-1.960	-2.301	-2.385	-4.228	-2.631	-1.799
	1st Diff.	-13.972	-14.372	-15.581	-9.029	-17.569	-16.040
	Level p-value	0.620	0.429	0.386	0.005	0.267	0.703
	1st Diff. p-value	0.000	0.000	0.000	0.000	0.000	0.000
PP	Level	-2.239	-2.710	-2.475	-3.191	-2.636	-1.882
	1st Diff.	-13.946	-14.457	-15.678	-15.187	-17.512	-16.086
	Level p-value	0.465	0.233	0.340	0.089	0.264	0.661
	1st Diff. p value	0.000	0.000	0.000	0.000	0.000	0.000
KPSS	Level	0.203	0.141	0.270	0.079	0.270	0.385
	1st Diff.	0.068	0.042	0.027	0.027	0.076	0.067

4.2 Symmetric cointegration

It is evident from table 4, which rejects the null hypothesis of no integration among all stock markets as shown by the Engle-Granger test. There is significant (5%) cointegration between the Pakistan stock market and all other stock markets. The cointegration between Pakistan and Russia is $|1.6692*|$, Brazil and Pakistan $|2.0898**|$, Pakistan and India $|1.9545**|$, China and Pakistan $|2.1315**|$ and South Africa and Pakistan $|2.0039**|$.

Table 4: Engle-Granger test

Pair	E-G ADF statistics	Cointegration
Brazil and Pakistan	-2.0898**	Yes
Russia and Pakistan	-1.6692*	Yes
India and Pakistan	-1.9545**	Yes
China and Pakistan	-2.1315**	Yes
South Africa and Pakistan	-2.0039**	Yes

TThe symbols ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

4.3 Asymmetric cointegration

The results of conventional and asymmetric cointegration among Pakistan and BRICS are reported using TAR and MTAR models. Table 6 and 7 presents estimation results of both conventional model and asymmetric model for all BRICS countries with Pakistan. In the discourse issue of serial correlation of residual series, an appropriate lag length is chosen. To begin with, a maximum of 12 lags is specified which will be selected later for each pair of markets to apply diagnostic analysis for example AIC, BIC, Ljung box Q statistics on residual series. Chan (1993) method adopted to estimate the threshold values represents a small difference of lag specifications on the selected values of the Threshold model.

Results reported in table 6 and 7 are consistent with TAR, MTAR models based on a low value of AIC and BIC statistics which is appropriate. It manifests that Russia Pakistan, Pakistan China, and Pakistan South Africa stock market pairs are cointegrated. It is inferred that the null hypothesis of no cointegration is rejected for three stock market pairs using MTAR models. The null hypothesis is rejected at 10% level of significance (ϕ = 4.458, 2.722, and 6.588) for three stock market pairs Russia Pakistan, Pakistan China, and Pakistan South Africa. The stock market pair: Pakistan South Africa (ϕ= 6.588) is also linked over the long run as the null hypothesis of co-integration is rejected at a significance level of 1%.

Finding of estimation results verify the existence of an asymmetric relationship rejecting the null hypothesis of symmetric adjustments $\rho_1 = \rho_2$ for BRICS and Pakistan stock market pairs. The null hypothesis of symmetric adjustments using MTAR is rejected for three stock market pairs: Russia Pakistan, Pakistan China, and Pakistan South Africa at 5% significance level (4.756), 10% significance level (3.069), and 1% significance level (9.086). It specifies that when the last equilibrium error has a different sign the speed of adjustment to equilibrium will be different. Due to this, the stock markets in our analysis are friction full. For this purpose, the researchers contend that symmetric cointegration models might provide a more accurate result and understanding of the relationship between equity markets (Anderson 1997).

4.3.1 Asymmetric cointegration between Pakistan and Brazil

Following the results in table 5, CTAR and CMTAR model has a low AIC statistic value of -321.077 and -322.029, BIC: -272.002 and -272.954 respectively. This lower value of AIC and BIC is considered to be the best. There is evidence of no cointegration between Pakistan and Brazil stock markets. The price adjustment is -0.077 for a decrease in price and -0.019 for an increase. The decrease in price (negative shock) from its equilibrium derived from the decrease in KSE 100 index is excluded at 7.7% per month while the increase in price (positive shock) from its equilibrium derived from an increase in the KSE-100 index is excluded at 1.9% per month. Pakistan's stock market takes almost 13 months to offset the effect of negative shocks (below threshold) while it takes almost 52 months to offset the effect of positive shock (above threshold). In the long-run adjustment between Pakistan and Brazil, we infer that both markets are not integrated. The investor has a safe portfolio and can opt for an opportunity in two portfolios.

Table 5: Asymmetric co-integration between Pakistan-Brazil

Item	Pakistan- Brazil			
	TAR	CTAR	MTAR	CMTAR
Estimate				
Lag	11	11	11	11
Threshold	0	-0.363	0	-0.082
Pos.coeff	-0.023	-0.014	-0.003	-0.019
Pos.t-value	(-1.056)	(-0.643)	(-0.127)	(-1.07)
Neg.coeff	-0.04*	-0.052**	-0.06***	-0.077**
Neg.t-value	(-1.822)	(-2.295)	(-2.739)	(-2.347)
Diagnostics				
AIC	-319.73	-321.08	-323.1	-322.03
BIC	-270.65	-272	-274.02	-272.95
LB test (4)	0.927	0.9	0.924	0.945
LB test (8)	0.992	0.987	0.9	0.992
LB test (12)	0.991	0.99	0.993	0.992
Hypothesis				
H1: no CI	2.115	2.767	3.75	3.229
H2:no APT	0.299	1.58	3.514	2.49
H2: p-value	0.585	0.21	0.062	0.116

The symbols ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively. QLB (p) represents the significance level for the Ljung-Box Q-statistic used to test serial correlation based on p autocorrelation coefficients (p = 4, 8, 12).

4.3.2 Asymmetric cointegration between Pakistan and Russia

Following the results in table 6, the CMTAR model's low AIC and BIC statistic values are -389.59 and -371.883 respectively which is appropriate for the model. There exist asymmetric cointegration among stock markets of Pakistan and Russia, which rejects the null hypothesis at a 10% significance level ($\phi = 4.458$). The decrease in price (negative shock) from its equilibrium derived from the decrease in the KSE 100 index is excluded at 9.7% per month. Similarly, the increase in price (positive shock) from its equilibrium derived from an increase in the KSE-100 index is excluded at 1.5% per month. This shows that it takes almost 10 months for the Pakistan stock market to offset the effects of negative shocks and 66 months to offset the effects of positive shock. It is evident when the last equilibrium error has a different sign, the speed of adjustment to equilibrium is different hence the stock markets are unresistant. The results show a very slow convergence for an increase in price (above threshold) while slightly high convergence for the price decrease.

Table 6: Asymmetric co-integration between Pakistan-Russia and India-Pakistan

Item	Pakistan- Russia				India Pakistan			
	TAR	CTAR	MTAR	CMTAR	TAR	CTAR	MTAR	CMTAR
Estimate								
Lag	2	2	2	2	2	2	2	2
Threshold	0	-0.454	0	-0.085	0	-0.402	0	-0.095
Pos.coeff	-0.029.	-0.019	-0.013	-0.015	-0.06*	-0.039	-0.046	-0.043*
Pos.t-value	(-1.464)	(-1.086)	(-0.616)	(-0.883)	(-1.764)	(-1.303)	(-1.359)	(-1.726)
Neg.coeff	-0.034	-0.078**	-0.05**	-0.097***	-0.054*	-0.081**	-0.066**	-0.133**
Neg.t-value	(-1.421)	(-2.372)	(-2.285)	(-2.867)	(-1.754)	(-2.313)	(-2.122)	(-2.34)
Diagnostics								
AIC	-384.83	-387.43	-386.28	-389.59	-393.73	-394.58	-393.92	-395.9
BIC	-367.12	-369.73	-368.58	-371.88	-376.02	-376.88	-376.21	-378.2
LB test (4)	0.901	0.877	0.891	0.839	0.902	0.951	0.925	0.931
LB test (8)	0.359	0.367	0.415	0.394	0.551	0.632	0.575	0.697
LB test (12)	0.378	0.371	0.436	0.456	0.599	0.667	0.621	0.692
Hypothesis								
H1: no CI	2.053	3.364	2.783	4.458*	2.999	3.43	3.093	4.098
H2:no APT	0.023	2.603	1.461	4.756**	0.018	0.861	0.202	2.166
H2:p-value	0.879	0.108	0.228	0.03	0.892	0.354	0.654	0.142

The symbols ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively. QLB (p) represents the significance level for the Ljung-Box Q-statistic used to test serial correlation based on p autocorrelation coefficients (p = 4, 8, 12).

4.3.3 Asymmetric cointegration between Pakistan and India

CTAR and CMTAR model has a low AIC statistic value of -394.584 and -395.902 and BIC is -376.878 and -378.196 respectively (table 6). It is evident from the results of CTAR: 3.43 and CMTAR 4.098 that there exists no integration among Pakistan and India's stock market. An investor can be benefited from diversification (Shahzad et al 2015). The price adjustment is -0.133 for the decrease in price while -0.043 for a price increase. The decrease in price (negative shock) from its equilibrium derived from the decrease in KSE 100 index is excluded at 13% per month. Similarly, the increase in price (positive shock) from its equilibrium derived from the increase in the KSE-100 index is excluded at 4.3% per month. The expectation is, Pakistan stock market takes almost 23 months to offset the effects of negative shocks (below threshold) while it takes almost 7 months to offset the effects of positive shock.

4.3.4 Asymmetric cointegration between Pakistan and China

From table 7, CMTAR: AIC and BIC statistic values are -154.922 and -140.742 and CTAR values are: -156.38 and -142.199 respectively. There exists a significant cointegration (ϕ=2.722) between Pakistan and China stock markets pair rejecting null hypothesis of symmetric cointegartion at 5% significance level. There is a price adjustment of -0.028 while -0.034 for the price increase. The decrease in price (negative shock) from its equilibrium derived from the decrease in the KSE 100 index is excluded at 2.8% per month. Similarly, the increase in price (positive shock) from its equilibrium derived from an increase in the KSE-100 index is excluded at 3.4% per month. This shows that it takes almost 35 months for the Pakistan stock market to offset the effects of negative shocks while it takes 29 months to offset the effects of positive shock. There is a slow convergence for an increase in price (above threshold) while slightly high convergence for the price decrease.

Table 7: Asymmetric co-integration between Pakistan and China

Item	Pakistan-China				Pakistan-South Africa			
	TAR	CTAR	MTAR	CMTAR	TAR	CTAR	MTAR	CMTAR
Estimate								
Lag	1	1	1	1	8	8	8	8
Threshold	0	-0.877	0	0.149	0	0.319	0	0.061
Pos.coeff	-0.012	0.004	-0.011	0.034	-0.03	-0.014	0.001	0.073.
Pos.t-value	(-0.636)	-0.264	(-0.623)	-1.035	(-0.957)	(-0.428)	-0.02	-1.579
Neg.coeff	-0.025.	-0.048***	-0.026.	-0.028**	-0.069**	-0.081**	-0.1***	-0.082***
Neg.t-value	(-1.505)	(-2.616)	(-1.525)	(-2.106)	(-2.038)	(-2.493)	(-3.042)	(-3.151)
Diagnostics								
AIC	-152.15	-156.38	-152.21	-154.92	-411.75	-413.29	-416.35	-420.19
BIC	-137.97	-142.2	-138.03	-140.74	-373.06	-374.59	-377.66	-381.5
LB test (4)	0.769	0.67	0.777	0.706	0.767	0.758	0.64	0.546
LB test (8)	0.524	0.416	0.563	0.725	0.935	0.935	0.849	0.795
LB test (12)	0.763	0.649	0.787	0.867	0.953	0.963	0.909	0.909
Hypothesis								
H1: no CI	1.329	3.46	1.359	2.722*	2.385	3.137	4.657	6.5888*
H2: no APT	0.307	4.531	0.369	3.069*	0.816	2.296	5.286	9.086***
H2: p-value	0.58	0.034	0.544	0.081	0.367	0.131	0.022	0.003

The symbols ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively. QLB (p) represents the significance level for the Ljung-Box Q-statistic used to test serial correlation based on p autocorrelation coefficients (p = 4, 8, 12).

4.3.5 Asymmetric cointegration between Pakistan and South Africa

The findings of table 7 show that CMTAR model has AIC and BIC statistic value of -420.194 and -381.502. There is significant asymmetric cointegration amongst Pakistan and South Africa's stock market with no diversification edge (ϕ= 6.588). The price adjustment is -0.082 for the decrease in price and 0.073 for an increase. The decrease in price (negative shock) from its equilibrium derived from the decrease in the KSE 100 index is excluded at 8.2% per month. Similarly, the increase in price (positive shock) from its equilibrium derived from the increase in the KSE-100 index is excluded at 7.3% per month. It takes almost 12 months for the Pakistan stock market to offset the effects of negative shocks while it takes 14 months to offset the effects of positive shock. Therefore there exists a very slight convergence for the price increase compared to the decrease in price.

4.4 Vector error correction model

4.4.1 Pakistan and Brazil

The previous model determines the relationship between Pakistan and Brazil stock markets. VECM model with threshold cointegration is estimated for the price adjustments between Pakistan and Brazil stock exchange. The results are shown in table 8. The value of AIC and BIC statistics is -546.783 and -476.115 for Brazil and Pakistan the statistic -531.718 and -461.05 is a better fit for Brazil as compared to Pakistan. Bivariate analysis among two variables is tested which is quite low R^2= 0.158 ignoring the impact of other variables. The R^2 statistic is 15.8% for Brazil and R^2 statistic is 15.8% for Pakistan. The null hypothesis of Granger causality between Pakistan Bazil is rejected at F statistic (3.227) at a 1% significance level. Pakistan stock market does have an impact on the Brazil market. There is a significant impact of previous prices for Brazil, stock market at 1% significance (F- statistic = 3.95). Short-term cointegration exists among Pakistan and Brazilian stock markets. The statistics for $\epsilon^+ = \epsilon^-$ 1.623 is insignificant with no asymmetric momentum equilibrium adjustments. For Brazil ϵ^+ is 0.014 while ϵ^- is 0.044 significant at 5% for positive shocks. Brazilian stock markets do not respond to short-term shocks while reacting to negative shocks at 44% (monthly). Pakistan stock market behaves in reverse and does not act in the short-run for both shocks (positive and negative). There is a unidirectional causality running from Brazilian stock markets to Pakistan stock markets.

Table 8: Vector error correction model for Pakistan-Brazil and Pakistan-Russia

	Pakistan-Brazil					Pakistan-Russia			
	Brazil. Est	Brazil.t	Pakistan. Est	Pakistan.t		Russia. Est	Russia.t	Pakistan. est	Pakistan.t
Constant	-0.005	-0.428	0.009	0.726	Constant	-0.008	-0.526	0.018.	1.548
ΔB^+_{t-1}	0.165	1.305	0.185	1.415	ΔR^+_{t-1}	0.325***	2.627	0.177*	1.782
ΔB^+_{t-2}	0.131	1.039	0.006	0.049	ΔR^+_{t-2}	-0.038	-0.321	-0.105	-1.096
ΔB^+_{t-3}	0.319**	2.505	-0.026	-0.2	ΔR^+_{t-3}	0.141	1.191	0.008	0.084
ΔB^+_{t-4}	-0.27**	-2.119	0.24*	1.833	ΔR^+_{t-4}	-0.112	-0.93	-0.031	-0.323
ΔB^-_{t-1}	0.007	0.067	-0.112	-1.018	ΔR^-_{t-1}	-0.029	-0.27	-0.045	-0.525
ΔB^-_{t-2}	-0.152	-1.432	0.468***	4.279	ΔR^-_{t-2}	0.118	1.045	0.405***	4.471
ΔB^-_{t-3}	-0.259**	-2.299	-0.083	-0.717	ΔR^-_{t-3}	-0.363***	-3.23	-0.096	-1.064
ΔB^-_{t-4}	0.37***	3.414	0.172.	1.543	ΔR^-_{t-4}	0.442***	3.812	0.237**	2.55
ΔP^+_{t-1}	0.046	0.386	0.144	1.172	ΔP^+_{t-1}	0.235.	1.502	0.143	1.139
ΔP^+_{t-2}	0.04	0.337	-0.107	-0.869	ΔP^+_{t-2}	0.211	1.369	-0.03	-0.246
ΔP^+_{t-3}	-0.076	-0.652	0.017	0.141	ΔP^+_{t-3}	0.151	0.971	-0.084	-0.67
ΔP^+_{t-4}	0.105	0.896	-0.109	-0.905	ΔP^+_{t-4}	-0.074	-0.48	-0.043	-0.353
ΔP^-_{t-1}	-0.072	-0.681	-0.027	-0.247	ΔP^-_{t-1}	0.106	0.783	-0.07	-0.643
ΔP^-_{t-2}	-0.021	-0.2	0.003	0.026	ΔP^-_{t-3}	-0.013	-0.092	-0.134	-1.229
ΔP^-_{t-3}	-0.042	-0.417	-0.095	-0.925	ΔP^-_{t-3}	0.017	0.129	-0.082	-0.779
ΔP^-_{t-4}	0.227**	2.266	0.007	0.066	ΔP^-_{t-4}	0.048	0.358	-0.085	-0.794
ϵ^+	0.014	1.208	0.012	1.044	ϵ^+	0.008	0.479	-0.015	-1.155
ϵ^-	0.044**	2.06	-0.019	-0.858	ϵ^-	0.079**	2.274	-0.018	-0.64
R^2	0.158	-	0.134	-	R^2	0.184	-	0.14	-
AIC	-546.783	-	-531.718	-	AIC	-421.44	-	-533.668	-
BIC	-476.115	-	-461.05	-	BIC	-350.773	-	-463	-
Q_{LB}-4	0.973	-	0.952	-	Q_{LB}-4	0.193	-	0.943	-
Q_{LB}-8	0.801	-	0.39	-	Q_{LB}-8	0.003	-	0.597	-
ΔB	3.227***	[0]	3.95***	[0]	ΔR	3.548***	[0]	3.571***	[0]
ΔP	1.264	[0.26]	0.455	[0.89]	ΔP	0.932	[0.49]	0.736	[0.66]
$\epsilon^+ = \epsilon^-$	1.623	[0.2]	1.603	[0.21]	$\epsilon^+ = \epsilon^-$	3.378*	[0.07]	0.009	[0.92]
$\alpha_1^+ \alpha_1^-$	0.48	[0.49]	0.404	[0.53]	$\alpha_1^+ \alpha_1^-$	0.587	[0.44]	0.871	[0.35]
$\Sigma\alpha_1^+ \Sigma\alpha_1^-$	1.061	[0.3]	0.011	[0.92]	$\Sigma\alpha_1^+ \Sigma\alpha_1^-$	0.587	[0.44]	0.871	[0.35]

4.4.2 Pakistan and Russia

Table 8 shows the value of AIC is -421.44 and -350.773 for Russia and Pakistan and BIC statistic is -533.668 and -463 which means that the overall model is a better fit for Pakistan as compared to Russia. The R^2 statistic is 18.4% for Russia and 14% for Pakistan. In bivariate analysis the relationship between two variables is tested, resulting in ignorance of other variables that can directly or indirectly affect the model. Therefore, the value of R^2 in case of bivariate analysis is low (Shahzad et al 2015). Three coefficients for Russia are significant at 1% and 5% and three coefficients are significant at level 1%, 5%, and 10% for Pakistan. The null hypothesis of the 'Granger causality' test between the Pakistan stock market and the Russian stock market is rejected as the F-statistic value is 3.571, significant at 1%. It is interpreted Pakistan stock price does impact Russian stock price whereas, previous prices for Russian stock also have an impact on spot prices for Russian stock as the F-statistics is 3.548, significant at 1%. It is evident from the findings that there exists short term integration between Pakistan and Russian stock markets.

The F-statistic for $\epsilon^+ = \epsilon^-$ is 3.378 with a significance level of 10%. Therefore, there exists 'momentum equilibrium adjustment' asymmetry. For Russia, ϵ^+ is 0.008 which is not significant while ϵ^- is 0.079 which is significant at 5% rendering for positive shock, the Russian stock market did not respond in the short-run while responding for a negative shock with 7.9% per month. However, the Pakistan stock market does not react even in the short-run for positive and negative shocks. It is inferred that there exists a bidirectional relationship in this pair of stock markets.

4.4.3 Pakistan and India

Following table 9, AIC and BIC statistic is -591.574 and -520.906 for India and -525.587 and -454.919 for Pakistan represents a better fit model for India as compared to Pakistan. The R^2 statistic is 13.5% for India and 11.5% for Pakistan. In the case of bivariate analysis the value of R^2 is low. The F-statistics of 'Engle-Granger causality' is 3.281, significant at level 1% which rejects the null hypothesis. The F-statistics value for $\epsilon^+ = \epsilon^-$ is 6.11 significant at 1%. Therefore, there exists a 'momentum equilibrium adjustment' asymmetry. For India ϵ^+ is 0.031 significant at 10% while ϵ^- is 0.136 significant at 1% representing positive and negative shock response in the Indian stock market in the short term.

Contrary to the Indian stock market, the equilibrium adjustment is not significant for the Pakistan stock market therefore it does not respond to a shock. There is evidence that unidirectional causality exists between Pakistan and India stock market pair.

Table 9: Error correction for Pakistan-India and China-Pakistan

	India. Est	India.t	Pakistan. Est	Pakistan.t		China. est	China.t	Pakistan. est	Pakistan.t
Constant	-0.006	-0.493	0.028**	2.005	Constant	-0.014	-1.006	0.028*	1.9
ΔI^+_{t-1}	0.106	0.851	0.073	0.518	ΔC^+_{t-1}	0.384***	3.139	-0.059	-0.447
ΔI^+_{t-2}	0.314**	2.535	0.117	0.832	ΔC^+_{t-2}	-0.1	-0.808	0.102	0.761
ΔI^+_{t-3}	-0.049	-0.402	-0.015	-0.107	ΔC^+_{t-3}	0.026	0.213	-0.183	-1.364
ΔI^+_{t-4}	-0.063	-0.524	-0.073	-0.534	ΔC^+_{t-4}	0.096	0.767	-0.064	-0.47
ΔI^-_{t-1}	0.074	0.667	0.027	0.211	ΔC^-_{t-1}	-0.123	-1.095	-0.088	-0.719
ΔI^-_{t-2}	-0.277**	-2.505	0.471***	3.741	ΔC^-_{t-2}	0.228**	2.058	0.435***	3.605
ΔI^-_{t-3}	0.094	0.8	0.089	0.67	ΔC^-_{t-3}	-0.02	-0.173	-0.045	-0.362
ΔI^-_{t-4}	0.161	1.375	0.352***	2.632	ΔC^-_{t-4}	0.195*	1.685	0.22*	1.75
ΔP^+_{t-1}	0.196*	1.84	0.126	1.042	ΔP^+_{t-1}	0.017	0.151	0.172	1.381
ΔP^+_{t-2}	0.107	0.99	-0.108	-0.875	ΔP^+_{t-2}	0.158	1.44	-0.03	-0.248
ΔP^+_{t-3}	-0.15	-1.398	-0.069	-0.564	ΔP^+_{t-3}	0.132	1.219	0.035	0.299
ΔP^+_{t-4}	0.012	0.116	-0.064	-0.521	ΔP^+_{t-4}	-0.029	-0.271	-0.048	-0.405
ΔP^-_{t-1}	-0.091	-0.943	-0.117	-1.064	ΔP^-_{t-1}	-0.016	-0.163	0.033	0.303
ΔP^-_{t-2}	0.014	0.144	-0.039	-0.364	ΔP^-_{t-2}	-0.093	-0.93	-0.003	-0.027
ΔP^-_{t-3}	-0.086	-0.943	-0.121	-1.164	ΔP^-_{t-3}	-0.002	-0.02	-0.089	-0.851
ΔP^-_{t-4}	0.017	0.191	-0.024	-0.237	ΔP^-_{t-4}	0.048	0.51	0.07	0.674
ϵ^+	0.031*	1.812	-0.006	-0.319	ϵ^+	-0.012	-0.793	0.013	0.788
ϵ^-	0.136***	3.39	0.003	0.064	ϵ^-	0.017***	2.607	-0.003	-0.434
R^2	0.135	-	0.113	-	R^2	0.124	-	0.122	-
AIC	-591.574	-	-525.587	-	AIC	-570.451	-	-528.341	-
BIC	-520.906	-	-454.919	-	BIC	-499.783	-	-457.673	-
Q_{LB}-4	0.927	-	0.629	-	Q_{LB}-4	0.983	-	0.944	-
Q_{LB}-8	0.808	-	0.02	-	Q_{LB}-8	0.965	-	0.011	-
ΔI	1.502	[0.16]	3.281***	[0]	ΔC	2.907***	[0]	3.649***	[0]
ΔP	1.17	[0.32]	0.659	[0.73]	ΔP	0.565	[0.81]	0.453	[0.89]
$\epsilon^+ = \epsilon^-$	6.1**	[0.01]	0.036	[0.85]	$\epsilon^+ = \epsilon^-$	3.18*	[0.08]	0.805	[0.37]
$\alpha^+_1 \alpha^-_1$	0.001	[0.98]	0.046	[0.83]	$\alpha^+_1 \alpha^-_1$	0.215	[0.64]	0.414	[0.52]
$\Sigma\alpha^+_1 \Sigma\alpha^-_1$	1.088	[0.3]	0.304	[0.58]	$\Sigma\alpha^+_1 \Sigma\alpha^-_1$	0.155	[0.69]	4.329**	[0.04]

4.4.4 Pakistan and China

From table 9, AIC and BIC statistic is -570.451 and -499.783 for China and -528.341 and -457.673 for Pakistan, the overall model is a better fit for China as compared to Pakistan. The R^2 statistic is 12.4% for China and 12.2% for Pakistan. In bivariate analysis the value of R^2 is low. There are three coefficients for China which are significant at 1% each while for Pakistan two coefficients are significant at 10% and 1%. The Granger causality test is used to estimate the lead lag short run relationship between stock market pairs. The null hypothesis of the 'Granger causality' test between Pakistan stock market and China stock market is rejected as F-statistic is 3.649 significant at 1%. This shows that Pakistan stock prices have an impact on China stock price whereas, previous prices for Chinese stock also have an impact on spot prices of Chinese stock as the F-statistic is 2.907 at 1% significance. There exists short-term integration between Pakistan and China. Moreover, the prices depend upon each other as there exists a significant relationship among markets. Hence the existence of unidirectional causality from Pakistan stock market to China stock market is inferred.

The null hypothesis of the 'Granger causality' test is accepted for Pakistan with the F-statistics: 0.565 and insignificant so, China stock market fluctuation does not affect the Pakistan stock market. The F-statistics for $\epsilon^+ = \epsilon^-$ is 3.18 at 10% significance with the evidence of 'momentum equilibrium adjustment' asymmetry. For China ϵ^+ is -0.012 which is not significant while ϵ^- is 0.017 at 1% significance. It is approved from the findings, for positive shock China stock market did not respond in the short term but responds for a negative shock with 1.7% per month. Similarly, for Pakistan, the F-statistic value $\epsilon^+ = \epsilon^-$ is insignificant. The F-statistic is insignificant for both with 0.013 for ϵ^+ and -0.003 for ϵ^-. Therefore, the Pakistan stock market does not react in short term to positive and negative shocks.

4.4.5 Pakistan and South Africa

Following the results in table 10, AIC and BIC statistic is -759.953 and -689.285 for South Africa and Pakistan: -529.672 and -459.005, representing overall model is a better fit for South Africa as compared to Pakistan. The R^2 statistic is 15.4% for South Africa and 12.7% for Pakistan. The value of R^2 bivariate analysis is low. Two coefficients for South Africa are significant at 1% and four at 10%, while for Pakistan two coefficients are significant at 10% and one is significant at 5%, and 10%.

The null hypothesis of the 'Granger causality' test between the Pakistan stock market and South Africa stock market is tested using F-statistics. The null hypothesis is rejected as the F-statistic value is 3.108, significant at 1%, showing Pakistan stock price does impact South Africa stock price whereas, previous prices for South African stocks have an insignificant impact on spot prices. For South Africa, F-statistic is insignificant at 0.954. Therefore, there exists short-term cointegration between Pakistan and South Africa, and the prices depend upon each other as there exists a significant relationship between both markets. Similarly, for Pakistan, the null hypothesis is rejected as the F-statistic value is 3.14 at a 1% significance level. South Africa stock price does cause a change in Pakistan stock price whereas previous prices for Pakistan stock do not have a significant impact on spot prices for Pakistan stock market due to insignificant F statistic (0.954). The F-statistics value for $\epsilon^+ = \epsilon^-$ is 0.982 and insignificant. For positive error correction term for South Africa stock price ϵ^+ is insignificant (0.006) while for negative error correction term the estimate ϵ^- is 0.033, significant at 5%.

From the analysis, the South African stock market only responds to negative shock with 3.3% per month. Similarly, the Pakistan stock market will react in short term for negative shocks at 3.9%. Therefore the existence of bidirectional causality exist between this pair.

Table 10: Error correction for Pakistan and South Africa

	South.Africa.est	South.Africa.t	Pakistan. Est	Pakistan.t
Constant	0.004	0.472	0.008	0.637
ΔS^+_{t-1}	-0.001	-0.006	0.464**	2.472
ΔS^+_{t-2}	0.202*	1.672	-0.205	-1.078
ΔS^+_{t-3}	0.213*	1.741	-0.156	-0.811
ΔS^+_{t-4}	0.03	0.245	-0.079	-0.406
ΔS^-_{t-1}	-0.06	-0.527	-0.326*	-1.823
ΔS^-_{t-2}	-0.206*	-1.786	0.713***	3.922
ΔS^-_{t-3}	-0.066	-0.549	0.069	0.365
ΔS^-_{t-4}	0.005	0.038	0.227	1.185
ΔP^+_{t-1}	0.048	0.613	0.221*	1.769
ΔP^+_{t-2}	-0.107	-1.372	-0.014	-0.115
ΔP^+_{t-3}	-0.014	-0.178	0.022	0.179
ΔP^+_{t-4}	0.003	0.038	0.039	0.335
ΔP^-_{t-1}	0.053	0.751	-0.063	-0.572
ΔP^-_{t-2}	0.21***	3.07	-0.001	-0.005
ΔP^-_{t-3}	-0.131*	-1.932	-0.068	-0.634
ΔP^-_{t-4}	0.219***	3.212	-0.032	-0.3
ϵ^+	0.006	0.272	0.053	1.411
ϵ^-	0.033**	2.499	-0.039*	-1.877
R^2	0.154	_	0.127	_
AIC	-759.953	_	-529.672	_
BIC	-689.285	_	-459.005	_
Q_{LB}-4	0.997	_	0.991	_
Q_{LB}-8	0.918	_	0.453	_
ΔS does not Granger cause	0.954	[0.47]	3.108***	[0]
ΔP does not Granger cause	3.14***	[0]	0.456	[0.89]
$\epsilon^+ = \epsilon^-$	0.982	[0.32]	4.782**	[0.03]
$\alpha^+_1 \alpha^-_1$	3.556*	[0.06]	0.155	[0.7]
$\Sigma\alpha^+_1 \Sigma\alpha^-_1$	3.089*	[0.08]	1.305	[0.26]

4.5 Multifractal de-trended fluctuation analysis

The analysis of stock market efficiency starts with the calculation of stock return for selected stock markets. Afterward, the slope of the generalized Hurst exponent is calculated for selected stock markets. Table 11 shows the results of the generalized Hurst exponent.

It is highlighted from the results that there is a slight change in generalized Hurst exponent due to the variation of $q(-4to4)$ except Pakistan. The multifractal behavior of stock prices is weaker as $h(q)$ moves from -4 to 4. This pattern of results depicts that the stock markets are becoming more efficient over time except in Pakistan. Large and small variations are the primary focus in the study which are measured through market deficiency measure 'MDM' (Mensi et al 2017).

Table 11: Generalized Hurst exponent

	Pakistan	India	Russia	South Africa	Brazil	China
-4	3.2947	0.6367	0.5712	0.6578	0.536	0.6964
-3	3.114	0.6241	0.5696	0.6473	0.5156	0.6785
-2	2.8928	0.5972	0.5578	0.6243	0.5016	0.6674
-1	2.3992	0.5756	0.5467	0.5975	0.4999	0.6278
0	0.6996	0.5438	0.5385	0.6042	0.5092	0.6193
1	0.591	0.5389	0.5299	0.6101	0.5187	0.5875
2	0.5697	0.5146	0.528	0.6187	0.5156	0.5701
3	0.5476	0.437	0.5213	0.6023	0.4965	0.5367

4.6 MDM ranking efficiency

Table 12: Ranking efficiency of Pakistan and BRICS

Rank	1997-2018		1997-2006		2007 to 2010	
	Country	MDM value	Country	MDM value	Country	MDM value
1	Brazil	0.0387	Russia	0.0694	Brazil	0.0576
2	Russia	0.0473	Brazil	0.8672	China	0.0666
3	China	0.0896	South Africa	0.9685	South Africa	0.7584
4	South Africa	0.1112	India	0.1123	Russia	0.1056
5	India	0.1278	China	0.1687	India	0.1289
6	Pakistan	1.2678	Pakistan	0.1879	Pakistan	0.5237

The ranking of efficiency using MF-DFA is presented in table 12, which depicts that Brazil's stock market is at the top of the efficiency ranking for selected stock markets. The MDM value for Brazil is 0.0387 for the whole sample period (1997-2018) and 0.0576 during the financial crisis (0.0576) which is close to zero. Here it depicts Brazil's stock market as more efficient during the financial crisis.

Russian stock market was efficient before the global financial crisis. The improved monetary system, improved financial liberalization, and regulatory mechanism are the factors for high market efficiency before the crisis. Russia's stock market lost its efficiency during the crisis period. High level of efficiency is maintained by the Brazilian stock market due to tight control and quick response to the information during the period of market uncertainty.

It is also expected that improved efficiency for an emerging market like Brazil (first) and China (second) is due to the privatization process and fast GDP growth rate (Boutchkova and Megginson 2000). Pakistan's stock market is inefficient in all cases which represents there is no impact of a crisis. Market development stages are important indicators of stock market efficiency (Chancharat et al 2007). Small market size is another key indicator for the low level of stock market efficiency. Therefore, market development and small market size could be the factors for a low level of efficiency for the Pakistan stock market.

5 Conclusion

This study examines asymmetric cointegration and ranking of efficiency between Pakistan and BRICS equity markets. Monthly data ranging from October 1997 to September 2018 has been used for analysis. Symmetric equilibrium association among Pakistan and BRICS is studied by Engle-Granger asymmetric cointegration tests (VECM) and asymmetric association by Threshold Autoregressive and Momentum Threshold Autoregressive model. The result of the study depicts that there exists no cointegration between the Pakistan-Brazil stock markets and the Pakistan-India stock markets, while there is evidence of cointegration among Pakistan-China and Russia-South Africa stock markets. In the short run, cointegration exists between of Pakistan-India, Pakistan-Russia, China, and South Africa however no integration between Brazil-Pakistan.

These findings help the institutional and regional investor to gain diversification benefit by investing in Brazil and Pakistan stocks as no cointegration exists. There is evidence of cointegration in long run among BRICS and Pakistan. The presence of long-term cointegration depicts that the impact of the financial crisis can be different for different stock markets (Shehzad et al., 2015). The speed of negative shocks is higher and significant for Pakistan Russia, Pakistan South Africa stock market pairs, where quick adjustment of positive shocks is higher as compared to negative shocks for Pakistan China. There is strong evidence of unidirectional causality running between Pakistan China and Pakistan Brazil pair, however, bidirectional causality runs from Pakistan Russia, Pakistan India, and Pakistan South Africa's stock markets. It recommends investors pay keen attention to the Pakistan stock market while investing in BRICS stock markets.

The results of MF-DFA (multifractal detrended fluctuation analysis) represents time varying efficiency of BRICS and Pakistan stock markets. We found Brazil stock market is more efficient as compared to other emerging markets in BRICS in global financial crisis. On the basis of MDM, Brazilian stock market performed well during the global financial crisis of 2007-2008, whereas, Pakistan's stock market is inefficient as compared to other selected stock markets. Russian stock tended to show high efficiency in the long run (before crisis) and in short run and these stock markets become less efficient in global financial crisis. Brazilian stock market maintained its efficiency due to tight control and quick response to new information in the stock market uncertainty period. Moreover, Brazil and China ranked higher due to privatization and a fast growth rate in gross domestic product.

Pakistan stock market is ranked lower and inefficient in uncertainty because most of the securities are held by local investors which do not highly affect the market. Market development stages are important indicators of stock market efficiency (Chancharat et al 2007). Small market size is another key indicator for the low level of stock market efficiency. Therefore, market development and small market size are the indicators of low level of efficiency for the Pakistan stock market.

The findings of this study approve market efficiency theory and portfolio theory which helps investors to benefit through diversification of different stocks. In the view of policymakers, market efficiency plays an important role in market growth through the proper distribution of wealth and the proper allocation of resources. The inefficient market is detrimental to the economic growth of a country. Therefore informed decisions and corrective measures can reduce the economic distortions of the country. It is evident from the findings of integration and efficiency that investors are encouraged to invest in Pakistan and Brazil stock markets to gain more diversification benefits due to no cointegration between the Brazil-Pakistan stock market. Brazil stock market is an efficient market among all the stock markets of BRICS therefore the chances of loss are less and hence investors are encouraged to invest in Pakistani and Brazilian stocks. The findings can be effectively implemented to improve asset allocation and portfolio rebalancing by the investment industry.

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