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On predictability in South Asian stock markets

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ABSTRACT

In this research I investigate persistence in monthly excess stock returns over risk free rates in two South Asian stock markets i.e. S&P CNX 500 and KSE 100 stock price indexes using non-Gaussian state space or unobservable component model with stable distributions and volatility persistence.

Results from non-Gaussian state space models show that both markets encompass volatility persistence. KSE 100 has a stable characteristic exponent of $\alpha$ 1.748, but for S&P CNX 500 index the value for the characteristic exponent $\alpha$ is 1.999 which shows normal behavior in this market. Both markets encompass persistent signal in returns at 10% level of significance. The efficiently estimated excess returns for S&P CNX 500 are 0.01% per month (0.12% per annum), and 0.015% per month (0.18% per annum) for KSE 100 index.

Key phrases: stock return predictability; unobserved components; fat tails; stable distributions

JEL Codes: C22, C53, G14

Word Count Main Text: 2434

1. INTRODUCTION

Fama (1991) shows that predictability in stock returns have been explored extensively in the literature. The motive behind exploring stock returns predictability is economic gains that could be attained due to suitable trading strategies (Xu 2004). However, researchers have focused on two aspects of empirical distribution of stock returns which they think are important for accurate predictability. For example, Akgiray and Booth (1986), Jensen (1991), de Vries (1991), Buckel (1995), Mantegna and Stanley (1995), and McCulloch (1997) found evidence of non-normality in stock returns. On the other hand Nelson (1991), Danielsson (1994), Fagan and Schwert (1990), Diebold and Lopez (1995), and Goose and Kroner (1995) found evidence of volatility persistence in stock returns.

Conard and Kaul (1988) employed state space or unobservable component model to predict stock returns considering that shocks in both the observation and state Equations are normal. Similarly, state space models have also been used by Harvey (1985) and Watson (1986) with the assumptions that the underlying errors are normal. However, McCulloch (1996a) and Bidarkota and McCulloch (2004) modeled stock returns to be non-Gaussian with fat tails.
In this study, I investigate possible existence of persistent predictable signal in monthly S&P CNX 500 and KSE 100 indexes excess returns over the respective risk free rates. In order to account for non-Gaussian data, I model returns within the framework of Parisian stable distributions that were also employed by Mantanga and Stanley (1995), Buckel (1995), and McCulloch (1997). Therefore, as in Oh (1994) and Bidarkota and McCulloch (1998), I relax normality assumption in favor of stable distributions because Kalman filter is not operable efficiently with stable distributions. Similarly, to explicitly account for volatility persistence in the return series I employ GARCH-like model. The motive behind this study is to find stock market predictability in KSE 100 index (Pakistani stock market) and compare it with neighboring south Asian stock market (S&P CNX 500: Indian Stock Market). The findings from this study are intended to help policymakers as well as business community in Pakistan.

The remaining paper is organized as follows. Section 2 outlines the most general model used in this paper, and some estimation issues. In section 3, I present data sources, empirical results, and hypotheses tests, and finally section 4 includes conclusions of the study.

2. STATE SPACE MODEL FOR STOCK RETURNS

In this research six versions state space or unobserved time series econometrics models are used. Model 1 is the most general model that encompasses unobservable component in stock returns including non-normal errors and GARCH-like effects which is shown in following thee Equation:

\[ r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim c_t z_t, \quad z_t \sim iid \quad S_\alpha (0, 1) \] (1a)

\[ (x - \mu) = \phi (x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_t c_{t-1} z_{2t} \]

\[ z_{2t} \sim iid \quad S_\alpha (0, 1) \] (1b)

\[ c_t^\alpha = \omega + \beta c_{t-1}^\alpha + \delta | r_{t-1} - E(r_{t-1} | r_1, ..., r_{t-2}) |^\alpha \\
+ \gamma d_{t-1} | r_{t-1} - E(r_{t-1} | r_1, ..., r_{t-2}) |^\alpha \] (1c)

Where,

\[ d_{t-1} = \begin{cases} 1 & \text{if } r_{t-1} - E(r_{t-1} | r_1, r_2, ..., r_{t-2}) < 0 \\ 0 & \text{otherwise} \end{cases} \]

Here \( r_t \) is the observed one-period excess return, \( x_t \) is an unobserved persistence components in the series, and \( Z_1 \) and \( Z_2 \) are independent white noise processes.
Model 2 is obtained restricting $\alpha = 2$ in model 1 which can be written as:

\begin{align*}
r_t &= x_t + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2\sigma_\varepsilon}z_t, \quad z_t \sim iid \, N(0, 1) \quad (2a) \\
(x_t - \mu) &= \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim \sqrt{2\sigma_\eta}c_tz_{2t}, \quad z_{2t} \sim iid \, N(0, 1) \quad (2b) \\
c_t^2 &= \omega + \beta c_{t-1}^2 + \delta |r_{t-1} - E(r_{t-1} | r_t, r_{t-1}, \ldots, r_{t-2})|^2 + \gamma d_{t-1} - E(r_{t-1} | r_t, r_{t-1}, \ldots, r_{t-2})|^2 \quad (2c)
\end{align*}

Setting $\beta = \delta = \gamma = 0$ in model 1 gives model 3, which is shown in Equation 3:

\begin{align*}
r_t &= x_t + \varepsilon_t, \quad \varepsilon_t \sim S_{\alpha}(0, c) \quad (3a) \\
(x_t - \mu) &= \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim S_{\alpha}(0, c, c) \quad (3b)
\end{align*}

When restricting $\phi = 0$ in model 1, the shocks $\varepsilon_t$ and $\eta_t$ are not separately identified so $c_t$ is also not identified. The resulting model is model 4 which is shown in Equations 4:

\begin{align*}
r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \sim c_tz_t, \quad z_t \sim iid \, S_{\alpha}(0, 1) \quad (4a) \\
c_t^\alpha &= \omega + \beta c_{t-1}^\alpha + \delta |r_{t-1} - \mu|^\alpha + \gamma d_{t-1} - |r_{t-1} - \mu|^\alpha \quad (4b)
\end{align*}

Where,

$$
\begin{cases}
d_{t-1} = \begin{cases}
1 & \text{if } r_{t-1} - \mu < 0 \\
0 & \text{otherwise}
\end{cases}
\end{cases}
$$

Model 5 shown in Equation 5 is obtained setting $\alpha = 2$ in model 4.

\begin{align*}
r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \sim \sqrt{2\sigma_\varepsilon}z_t, \quad z_t \sim iid \, N(0, 1) \quad (5a) \\
c_t^2 &= \omega + \beta c_{t-1}^2 + \delta |r_{t-1} - \mu|^2 + \gamma d_{t-1} - |r_{t-1} - \mu|^2 \quad (5b)
\end{align*}

Restricting $\beta = \delta = \gamma = 0$ in model 4 results in model 6 which is presented in Equation 6:

\begin{align*}
r_t &= \mu + \varepsilon_t, \quad \varepsilon_t \sim S_{\alpha}(0, c) \quad (6)
\end{align*}

A random variable $x$ will have stable distribution $S_{\alpha}(0, c)$ when its log characteristic function can be represented as:

$$
\ln E \exp(i\eta x) = i\eta c - |c_t|^\alpha
$$

(7)
The parameter $c > 0$ measures scale whereas the parameter $\delta(-\infty, \infty)$ measures location and $\alpha \in (0, 2]$ is the characteristic exponent that governs the tail behavior. A small value of $\alpha$ indicates thicker tail and normal distribution pertaining to symmetric stable family encompassing $\alpha = 2$ whose variance is equal to $2c^2$.

In the process contained in Equation (1c) we restrict $\omega > 0, \beta > 0, \delta > 0$, and $\gamma \geq 0$. The theoretical term involving dummy variable $d_{t-1}$ captures leverage effects that are transmitted from negative shock to increase in future volatility more than a positive shock of equal magnitude (Nelson 1991, and Hamilton Susmel 1994). Abstracting from the threshold term, when the errors are normal, the model of volatility persistence reduces to GARCH-normal process.

Any predictable variation in excess return is because of persistent component $\chi_\tau$, which are assumed to follow a simple AR (1) process. When predictable component in Equation 1 becomes significant, than $E(\tau_t | \tau_1, \ldots, \tau_{t-1})$ provides a useful forecast of returns. However, when $c_{t-n}$ and $\phi$ or one of these is negligible, the returns are purely random, so these may display spurious predictions.

2.1. Estimation Issues

Non-Gaussianity of the state space model in Equation 1 creates complication in estimation even without the presence of conditional heteroskedasticity. This happens because the Kalman filter is no longer optimal due to the non-Gaussian nature of shocks.

The general recursive-filtering algorithm due to Sorenson and Alspach (1971) provides optimal filtering and predictive densities under any distribution for the errors and the formula for computing the log likelihood function. These formulae are presented in Appendix-A.

The recursive equation that is employed to compute filtering and predicting densities are given in the form of integrals whose close form analytical expressions are generally intractable, especially in very special cases. In this study, I numerically evaluate these integrals.

Stable distribution and density may be evaluated using Zolotrov’s (1986) proper integral representation or by taking inverse Fourier transformation of the characteristic function. However, McCulloch (1996b) has developed a fast numerical approximation to stable distribution and density that has an expected relative density of the precision of $10^{-6}$ for $\alpha \in [0.84, 2]$. I, therefore, restrict $\alpha$ in this range for computational convenience.

Lumsdaine (1996) shows that the effect of initial values in GARCH volatility process on the properties of the parameter estimates in GARCH (1, 1) is asymptotically negligible. Diebold and Lopez (1995) suggests to set the initial conditional variance ($2c_0^2$ where it exists) equal to sample variance at the first iteration and the subsequent
iterations to sample variance from simulated realizations with estimated parameters (from the previous iterations). Engle and Bollerslev (1986) suggests initializing the GARCH process using estimates of $C_0$ unconditional value obtained from the volatility process in Equation 1c.

3. EMPIRICAL RESULTS

3.1 Data Sources

Monthly excess returns in two South Asian stock markets over relevant risk free rates are employed from March 1991 through February 2004. Monthly stock prices for Pakistani stock market index KSE 100 and Indian stock market index S&P CNX 500 were obtained from Datastream. Pakistan Treasury bill rates and Indian central bank discount rates are the two risk free rates that were obtained from September 2004 version of International Financial Statistic (IFS) CD-ROM. Excess returns are expressed as percent per month throughout the study. Figure 1 plots excess return series for S&P CNX 500 index and Figure 2 plots excess return series for KSE 100 index. The plots shown in these Figures encouraged me to employ models for detecting possible persistence of mean returns using state space or unobserved component model of stock returns.

3.2 Estimation Results

Tables 1 and 2 show the estimation results for S&P CNX 500 and KSE 100 stock price index for different models estimated for this study. These Tables show parameter estimates for characteristic exponent $\alpha$, volatility persistence parameter $\beta$, ARCH parameter $\sigma$, leverage parameter $\gamma$, signal to noise ratio $C_q$, and AR coefficient of persistent component of returns $\phi$. There is ample difference in two markets that can be characterized by low values of characteristic exponent $\alpha$, the volatility persistence parameter $\beta$, and high values of leverage parameter $\gamma$ in KSE 100 index compared to S&P CNX 500. The predictable component in KSE 100 stock price index is statistically significant whereas there is less evidence of statistically significant predictable component in S&P CNX 500.

Figures 3 and 4 show filter mean $E(x_t | r_1, r_2, r_3, \ldots, r_{n})$ for Indian and Pakistani stock markets respectively. These plots show that predictable component appear to be constant which indicates that variation in its parameter estimates might not be component in forecasting excess returns.

3.3 Hypotheses Test

In the following sub-sections the tests for normality, volatility persistence, and persistence in mean returns are elaborated. All tests are based on likelihood ratio test statistics.
3.3.1 Test for Normality

This test is based on the null of $\alpha = 2$ in model 1. The LR test statistics for this test has non-standard distribution because the null hypothesis lies on the boundary of the admissible values for $\alpha$; therefore, standard regularity conditions are not satisfied. Inferences are therefore derived from test statistics based on the critical values due to McCulloch (1997).

Based on LR test statistics, the null hypothesis for KSE 100 index can easily be rejected at a significance level of better than 0.005 using critical values from McCulloch (1997). However, using LR test statistics, the null hypotheses of normality, and no volatility persistence for S&P CNX 500 index is not rejected. The study results indicate that even after accounting for GARCH-like behavior, the excess returns are significantly non-normal.

3.3.2 Test for Volatility Persistence

The test for no volatility persistence (homoskedasticity) can be formulated by restricting $\beta = \delta = \gamma = 0$ in model 1. The statistical inferences for this test are based on $\chi^2$ distributions.

The LR for the null of no GARCH which is to test $\beta = \delta = \gamma = 0$ reported in Tables 1 and 2 respectively for S&P CNX 500 and KSE 100 indexes. Homoskedasticity in both the markets is strongly rejected with $\chi^2$ critical values.

3.3.3 Test for Persistence in Mean Returns

The null hypothesis for this test is obtained from setting $\phi = 2$ in model 1 which assumes that return series are random. In this case the standard likelihood ratio test statistics for this test are not applicable because the two shocks $\xi_t$ and $\eta_t$ are not separately identified so the scale ratio $c_\eta$ is also not identified. Similarly, the bound for the asymptotic distribution of a standardized likelihood ratio test statistics due to Hansen (1992) which is applicable in such cases may result in under-rejection of the null or a subsequent power loss as noticed by Hansen himself. In addition, the test statistics is computationally very intense especially for this study, so I refrain from using it. Therefore, the inferences reported are based on the critical values obtained from $\chi^2$ and $\chi^2$ distributions.

Based on the LR test statistics for S&P CNX 500 index, the null hypothesis of no persistence in mean returns ($\phi = c_\eta = 0$) is rejected at 10 percent level of significance using critical values from $\chi^2$ distribution. However, the null can not be rejected using critical values from $\chi^2$ distribution at 10% level of significance. For KSE 100 stock price index, the null is rejected at 10% level of significance using critical values from $\chi^2$ or even $\chi^2$ distribution. Therefore, after accounting for normality and volatility persistence there exist statistically significant persistent signals in both the markets.
3.3.4 Additional Tests on Normality and Volatility Persistence

The tests for non-normality and volatility persistence are repeated considering model 4 as an alternative model. In this case model 5 is null model for non-normality and model 6 for homoskedasticity. The intuition behind these additional tests is to find the impact of excluding predictable component (from state space model) on the inferences from our model.

LR test statistics for normality and volatility persistence are reported in last two rows of column 5 in Tables 1 and 2 respectively for S&P CNX 500 index and KSE 100 index. Based on these results we failed to reject hypotheses of normality and no volatility persistence for S&P CNX 500 index. However, we reject both normality and no volatility persistence component in KSE 100 index. Figures 5 and 6 plot scales from model 4 for S&P CNX 500, and KSE 100 index respectively which show the evidence of highly non-constant scales in these markets, however, scales for KSE 100 index shows spikes in different time periods exhibiting a tendency towards stock market crashes.

3.3.5 Test for Leverage Effect

Absence of leverage effect imply that negative shock do not necessarily lead to negative increase in future volatility than positive shocks of the same magnitude. This hypothesis can be tested setting $\gamma = 0$, with $\gamma > 0$ as alternate hypothesis that the leverage effect exists. The results (not reported for brevity) strongly reject the null hypothesis in favor of leverage effects both for S&P CNX 500 and KSE 100 index.

3.4 Discussions on Results

The study results on hypothesis tests reveal that monthly KSE100 index excess returns from March 1991 through February 2004 do posses significant non-normality that is predictable even after accounting for conditional heteroskedasticity. Similarly, volatility persistence is also statistically significant. Leverage effects in volatility is insignificant, however, there is an evidence of statistically significant predictability at 10% level using critical values from $\chi^2$ as well as $\chi^2$ distribution. Statistically significant evidence of volatility 14 persistence exists in S&P CNX 500 index but there is no evidence of non-normality in this market. The state space models do not show statistically significant persistent signal in return series after taking into account volatility persistence at 10% level of significance using critical values $\chi^2$ from distribution, but we are not able to reject the null of no persistent signal in returns using critical values from $\chi^2$ distribution for this market at 5% level of significance.

As shown in Figure 6 KSE 100 index show highly non-constant scales and when compared to S&P CNX 500 index (Figure 5), the Figure shows random spikes in the years 1992, 1998, and 2000-2001. The plausible cause of these spikes are the external events during these years e.g. Gulf War, Asian Financial Crises, and crises after...
September 11, 2001 respectively that sensitized stock market agents that caused in instability in the stock prices and hence the market. Policymakers might be interested to address underlying policy issues enhance investor’s confidence that would strengthen the stock markets in the country and help stabilize the economy.

4. CONCLUSION

In this study non-Gaussian state space or unobserved component models are employed to find possible predictability in two South Asian stock markets (S&P CNX 500 and KSE 100). The state space models fully account for non-normality and volatility persistence that might be present in return series. S&P CNX 500 index demonstrate estimated value of characteristic exponent $\alpha$ which is close to normal behavior, however, the excess return encompass ample evidence of stock return volatility characterized by GARCH-like behavior. KSE 100 excess stock returns demonstrate significant leptokurtosis. The estimated value of characteristic exponent is well away from the value pertaining to normal behavior in this market, and excess stock returns exhibit ample persistence in stock return volatility that can be characterized by a GARCH-like process. There is insignificant leverage effect in the stock return volatility in both the markets (S&P CNX 500, and KSE 100) indicating that the negative shocks do not necessarily lead to greater increases in future volatility than the positive shocks of the equal magnitude. Unlike S&P CNX 500, KSE 100 index exhibits scale spike that shows a tendency towards stock market crashes. Policymakers might be interested to address underlying policy issues that might help improving the major stock market of the country.

The study results on predictability of monthly stock returns are statistically significant in KSE100 index but such results are less significant for S&P CNX 500 index. The efficiently estimated excess returns for S&P CNX 500 are 0.01% per month (0.12% per annum), and 0.015% per month (0.18% per annum) for KSE 100 stock price index.

Appendix A: Sorenson-Alspach Filtering Equations

Let $Y_t$, $t = 1, \ldots, T$, be an observed time series and $X_t$ an unobserved state variable, stochastically determining $Y_t$. Denote $Y_t = \{Y_1, \ldots, Y_T\}$. The recursive formulae for obtaining one-step ahead prediction and filtering densities, due to Sorenson and Alspach (1971), are as follows:

$$p(x_t | Y_{t-1}) = \int_{-\infty}^{\infty} p(x_t | x_{t-1}) p(x_{t-1} | Y_{t-1}) dx_{t-1}, \quad (A1)$$

$$p(x_t | Y_t) = p(y_t | x_t) p(x_t | Y_{t-1}) / p(y_t | Y_{t-1}), \quad (A2)$$

$$p(y_t | Y_{t-1}) = \int_{-\infty}^{\infty} p(y_t | x_t) p(x_t | Y_{t-1}) dx_t. \quad (A3)$$
Finally, the log-likelihood function is given by:

$$\log p(y_1, \ldots, y_T) = \sum_{t=1}^{T} \log p(y_t | Y_{t-1}).$$

(A4)

REFERENCES


### Table 1: Model Estimates (for S&P CNX500 Excess Returns) with Leverage Effects

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<th>Parameters</th>
<th>Model 1</th>
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<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
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Notes on Table 1

1. The following unobserved component or state space model with non-normality (stable model) is employed to estimate the results shown in the table.

\[ r_t = x_t + \varepsilon_t, \quad \varepsilon_t \sim c z_{1t}, \quad z_{1t} \sim \text{iid } S_\alpha(0,1) \]  

\[ (x_t - \mu) = \phi(x_{t-1} - \mu) + \eta_t, \quad \eta_t \sim c_{\eta} z_{2t}, \quad z_{2t} \sim \text{iid } S_\alpha(0,1) \]  

(1a)

(1b)

2. All estimates are rounded off to the third decimal place.

3. Hessian-based standard errors for the parameter estimates are reported in parentheses. LR (\( \hat{\phi} = \hat{c}_{\eta} = 0 \)) gives the value of the likelihood ratio test statistic. It is a test for no predictable components in excess returns. Under this null, the distribution of the LR test statistic is non-standard (see section 3.2 in the text for an elaboration).

4. P-values generated by estimating Gaussian versions of Models 1 and 2 with data simulated from the estimated Gaussian Model 2 are reported in parentheses.

5. LR (\( \alpha = 2 \)) gives the value of the likelihood ratio test statistic for the null hypothesis of normality.

6. The small-sample critical value at the 0.01 significance level for a sample size of 300 is reported to be 4.764 from simulations in McCulloch (1997).

7. LR (\( \beta = \delta = \gamma = 0 \)) is the test for no volatility persistence. This test is evaluated at \( \chi^2_1 \) p-values.
Table 2: Model Estimates (for KSE100 Excess Returns) with Leverage Effects

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<th>Parameters</th>
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<th>Model 4</th>
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<td>(0.000)</td>
<td>(0.007)</td>
<td>(0.006)</td>
<td>(0.011)</td>
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<td>$\beta$</td>
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<td>0.014</td>
<td>0.031</td>
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<td>(0.000)</td>
<td>(1.079)</td>
<td>(0.468)</td>
<td>(0.946)</td>
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<td>$\delta$</td>
<td>0.002</td>
<td>6.45e-13</td>
<td>2.85e-13</td>
<td>8.58e-13</td>
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<td>(0.005)</td>
<td>(8.46e-10)</td>
<td>(1.06e-10)</td>
<td>(2.36e-9)</td>
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<td>$\gamma$</td>
<td>18.170</td>
<td>0.541</td>
<td>1.66e-9</td>
<td>3.038e-10</td>
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<td></td>
<td>(5.916)</td>
<td>(0.442)</td>
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<td>(3.20e-7)</td>
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<td>$\epsilon_\gamma$</td>
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<td>1.25e-10</td>
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<td>(6.77e-7)</td>
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<tr>
<td>$\epsilon$</td>
<td>0.001</td>
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<td>0.016</td>
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<td></td>
<td>(0.003)</td>
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<td></td>
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<td>(0.012)</td>
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<tr>
<td>$\phi$</td>
<td>0.191</td>
<td>0.613</td>
<td>0.156</td>
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<tr>
<td></td>
<td>(0.070)</td>
<td>(0.313)</td>
<td>(0.001)</td>
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<td>Log L</td>
<td>88.494</td>
<td>83.738</td>
<td>85.41</td>
<td>82.529</td>
<td>79.792</td>
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<td>LR ($\alpha = 2$)</td>
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<td>LR ($\beta = \delta = \gamma = 0$)</td>
<td>6.166</td>
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<tr>
<td>LR ($\phi = \epsilon_\gamma = 0$)</td>
<td>11.930</td>
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Notes on Table 2.
1. See notes in Table 1.
Figure 1 – Monthly SnP CNX 500 excess stock returns

Figure 2 – Monthly KSE 100 excess stock returns
Figure 3 – SnP CNX 500 excess returns and filter estimates

Figure 4 – KSE100 excess returns and filter estimates